60 100

10.1

Fermat's Principle: a light ray chooses the path between two points that minimizes the time to travel between them. I'll aplly this to two points on either side of a dielectric interface to derive Snell's law.

Consider two points on either side of a dielectric interface. Points r_1 is in medium 1 with refractive index n_1, and point r_2 is in medium 2 with refractive index n_2. Thee refractive index is at $y=0$.

The total time needed to travel from r_1 to r_2 is:

$$T = \frac{r_1}{v_1} + \frac{r_2}{v_2}$$

where v_1 and v_2 are the speeds of light in respective media.
Since $v = c/n$ where c is the speed of light in vacuum, we can rewrite this as:

$$T = \frac{n_1}{c}\sqrt{(x_1-x_2)^2+z_1^2} + \frac{n_2}{c}\sqrt{(x_2-x)^2+z_2^2}$$

Here, x is the x-coordinate of the point where the ray crosses the interface, which is the parameter we need to find to minimize the travel time.

To find the minimum travel time, I'll differentiate with respect to x and set the result equal to zero:

$$\frac{dT}{dx} = \frac{n_1}{c}\frac{(x-x_1)}{r_1} - \frac{n_2}{c}\frac{(x_2-x)}{r_2} = 0$$

This can be rewritten as:

$$\frac{n_1 \sin\theta_1}{c} = \frac{n_2 \sin\theta_2}{c}$$

where θ_1 and θ_2 are the angles between the ray and the normal to the interface in each medium. Simplifying :-

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This is Snell's Law, which describes how light refracts when passing from one medium to another. The physical interpretation is that the ratio of the sines of the angles of incidence and refraction is equal to the reciprocal of the ratio of the refractive indices of the two media

10.2a Frensel Equations for Reflectivity and Transmissivity

To find the reflectivity and transmissivity of a dielectric interface, I'll use Frensel's equations and the Poynting vector to find the ratios of incoming and outgoing energy.

The Poynting vector represents the energy flux of an electromagnetic wave:

$$\vec{P} = \vec{E} \times \vec{H} = \vec{E} \times \sqrt{\frac{\epsilon}{\mu}}\, \hat{k} \times \vec{E} = \sqrt{\frac{\epsilon}{\mu}}\, E^2 \hat{k}$$

The ratio of the reflected to the incoming energy is:

$$R = \frac{|E_r|^2}{|E_0|^2}$$

And the ratio of the transmitted to the incoming energy is:

$$T = \frac{n_2 |E_t|^2}{n_0 |E_0|^2}$$

There are two cases to consider:

1. $\vec{E}$ perpendicular to plane of incidence (s-polarization):

In this case, the electric field is perpendicular to the plane containing the incident, reflected, and transmitted rays. From the boundary conditions at the interface, we can derive:

$$R = \frac{|E_r|^2}{|E_0|^2} = \frac{\sin^2(\theta_2 - \theta_t)}{\sin^2(\theta_i + \theta_t)}$$

And :

$$T = \frac{n_2 |E_t|^2}{n_0 |E_0|^2} = \frac{4 n_1 n_2 \cos^2\theta_i \sin^2\theta_t}{n_0 \sin^2(\theta_i + \theta_t)}$$

Simplifying

$$T = \frac{4 \sin\theta_i \cos\theta_i \sin\theta_t \cos\theta_t}{\sin^2(\theta_i + \theta_t)}$$

2- $\vec{E}$ in the plane of incidence (p-polarization):

In this case, the electric field lies in the plane containing the incident, reflected, and transmitted rays.

From the boundary conditions,

$$R = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

And :

$$T = \frac{4 \sin\theta_i \cos\theta_i \sin\theta_t \cos\theta_t}{\sin\theta \sin(\theta_i + \theta_t) \cos^2(\theta_i - \theta_t)}$$

These equation describes how the intensity of light is divided between reflection and transmission at a dielectric interface, depending on the polarization, angle of incidence, and refractive indices.

10.2 c

The Brewster angle is the angle of incidence at which light with a particular polarization is perfectly transmitted through a dielectric surface, with no reflection. This occurs when the reflected and refracted rays are perpendicular to each other.

For light travelling from a medium with refractive index n_1 to a medium with refractive index n_2, the Brewster angle θ_B is given by:-

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

For an air-to-glass interface (air → glass) with $n_1 = 1$ and $n_2 = 1.5$:

$$\theta_B = \tan^{-1}\left(\frac{1.5}{1}\right) = 56.3^\circ$$

For a glass-to-air interface (glass → air):

$$\theta_B = \tan^{-1}\left(\frac{1}{1.5}\right) = 33.7^\circ$$

The output windows in gas laser tubes are often tilted at this angle to define the direction of polarization for the laser beam. This is because at the Brewster angle, only light polarized perpendicular to the plane of incidence is reflected, while light polarized ~~to~~ parallel to the plane of incidence is completely transmitted.

$\boxed{10.2\,d}$

The critical angle is the angle of incidence beyond which total internal reflection occurs. Total internal reflection happens when light travelling from a medium with a higher refractive index to one with a lower refractive index reaches an angle where no light is refracted across the boundary.

Using Snell's Law, the critical angle θ_c is given by:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This is only defined when $n_1 > n_2$, i.e, when light is going from a higher to a lower index of refraction. For a glass-to-air interface with $n_1 = 1.5$ and $n_2 = 1$:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$$

Total internal reflection is possible only when going from a higher to a lower index of refraction. This phenomenon is used in fibre optics, prisms, and other optical devices to guide light along specific paths without loss due to refraction.

10.3a

Consider a wave at a normal incidence to a dielectric layer with index n_2 between layers with indices n_1 and n_3, as shown in Fig A1.13.

At each interface, the ratio of the reflected and the incoming field strength is given by:

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

And the ratio of the transmitted and the incoming field strength is:

$$t_{12} = \frac{2n_1}{n_1 + n_2}$$

When light passes through the middle layer of thickness d, it undergoes a phase change:

$$\phi = e^{ik_2 d}$$

where k_2 is the wave number in the medium with index n_2.

If the incoming wave has magnitude E^+, the total reflected wave E^- will be an infinite sum of reflections between the interfaces:

$$E^- = E^+ \left[r_{12} + t_{12} r_{23} e^{ik_2 d} t_{21} + t_{12} r_{23}{}^{ik_2 d} r_{21} r_{23} e^{ik_2 d} t_{21} + \dots \right]$$

This can be simplified using the geometric series Formula

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \text{ for } |x| < 1 :$$

$$E^- = E^+ \left[r_{12} + \frac{t_{12} r_{23} e^{ik_2 d} t_{21}}{1 - r_{21} r_{23} e^{ik_2 d}} \right]$$

Since $r_{12} = -r_{21}$ and $t_{12} t_{21} = 1 - r_{12}^2$, we can further simplify :

$$E^- = E^+ \left[\frac{r_{12} + r_{23} e^{ik_2 d}}{1 + r_{12} r_{23} e^{ik_2 d}} \right]$$

Therefore, the total reflection coefficient is:

$$R = \frac{|E^-|^2}{|E^+|^2} = \left| \frac{r_{12} + r_{23} e^{ik_2 d}}{1 + r_{12} r_{23} e^{ik_2 d}} \right|^2$$

This equation describes how the reflectivity depends on the refractive indices of all the three layers and the thickness of the middle layer. The exponential term represents the phase change due to the optical path length in the middle layer, which can lead to constructive or destructive interference between the multip reflections.

10.3b Conditions for zero Reflection

To find values for n_2 and d such that the reflection vanishes, I need to determine when the numerator in the reflection coefficient becomes zero.

$$r_{12} + r_{23}e^{ik_2d} = 0$$

Since r_{12} and r_{23} are real numbers, for the numerator to vanish:

$$e^{ik_2d} = -1 \Rightarrow k_2d = \pi$$

This means:

$$\frac{2\pi}{\lambda_2} d = \pi \Rightarrow d = \frac{\lambda_2}{4}$$

where λ_2 is the wavelength in the medium with index n_2.
This is the quarter-wavelength condition for an antireflection coating. The thickness of the coating should be one-quarter of the wavelength in that material.

Additionally, for the reflection to vanish completely, we need:

$$r_{12} = -r_{23}$$

Substituting the expressions for the reflection coefficients:

$$\frac{n_1 - n_2}{n_1 + n_2} = -\frac{n_2 - n_3}{n_2 + n_3}$$

Solving this equation:

$$\frac{n_1 - n_2}{n_1 + n_2} = \frac{n_3 - n_2}{n_2 + n_3}$$

After algebric manipulation:

$$n_1n_3 + n_1n_2 - n_2^2 - n_2n_3 = n_1n_3 - n_1n_2 + n_2^2 - n_2n_3$$

Simplifying

$$2n_1n_2 - 2n_2^2 = 0 \Rightarrow n_2 = \sqrt{n_1n_3}$$

Therefore, there will be no reflection.

$\boxed{10.59}$ The divergance angle of a laser beam is given by:

$$\theta = \frac{\lambda}{\pi D_{nw}}$$

where λ is wavelength of the laser and D_{nw} is the diameter of the beam waist (narrowest point).

For a laser with wavelength $\lambda = 790nm$ and beam waist diameter $D_{nw} = 1\mu m$:

$$\theta = \frac{790 \times 10^{-9}}{\pi . 1 \times 10^{-6}} = 0.251 \text{ rad}$$

Converting to degrees:

$$\theta = 0.251 \text{ rad} \times \frac{180°}{\pi} = 14.4°$$

This calculation shows hows the divergence angle depends on the wavelength and beam diameter, which is importan for understanding the spreading of laser beams over distance.

$\boxed{10.5b}$ To find the new wavelength λ_{new} when the beam passes through an aperture of diameter D, I can use the relationship :

$$\lambda_{new} = \lambda \cdot \frac{D_{new}}{D}$$

where D_{new} is the new beam diameter.

Given $\lambda = 790\,nm$, and $D_{new} / D = 10^{-7} / 10^{-6} = 0.1$:

$$\lambda_{new} = 790nm \cdot 0.1 = 79nm$$

This calculation shows how the effective wavelength changes when a beam passes through an aperture, which is important in understanding diffraction effects in optical systems.

iii understanding diffraction effects in optical systems

10.5c

The angular resolution of an optical system is limited by diffraction and is given by:

$$\theta = \frac{\lambda}{\pi \cdot D}$$

where λ is the wavelength of light and D is the aperture diameter.

For visible light with $\lambda = 600nm$ and an aperture (pupil) diameter of $D = 1\,cm$:

$$\theta = \frac{600 \times 10^{-9}}{\pi \cdot 1 \times 10^{-2}} = 1.9 \times 10^{-5} \,rad$$

At a distance of $L = 200km$, the smallest resolvable detail would be:

$$r = L \cdot \theta = 200 \times 10^3 m \times 1.9 \times 10^{-5} rad = 3.8m$$

This means that at a distance of 200km, the smallest detail that can be resolved is about 3.8 meters, which is much larger than a license plate. Therefore, it would not be possible to read a licence plate from this distance.